

**Lecture 16 - March 16**

**Reactive System: Bridge Controller**

## Announcements

- **ProgTest1** result to be released by the end of Friday
- **Lab3** released
- **Example Questions** for Written Test 2 released 
- To be completed by the final exam:  
**Makeup lectures** for WT1, WT2, ProgTest1, ProgTest2

# Lecture

## Reactive System: Bridge Controller

***First Refinement: Invariant Preservation  
Concrete, Refined Events***

"dd" events, existing in both m0 and m1.

# PO/VC Rule of Invariant Preservation: Sequents

xx  
 $\delta = 0 \vee \ell = 0$   
 $a$   
 $c-1$   
 $a=0 \vee$   
 $c-1=0$

## Abstract m0

variables: $n$	ML_out when $n < d$ then $n := n + 1$ end	ML_in when $n > 0$ then $n := n - 1$ end
invariants: inv0.1: $n \in \mathbb{N}$ inv0.2: $n \leq d$	$n' = n + 1$ $n := n + 1$	

$A(c) \rightarrow$  axioms  
 $I(c, v) \rightarrow$  abstract inv.  
 $J(c, v, w) \rightarrow$  concrete inv.  
 $H(c, w)$  concrete guard.  
 $\vdash$   
 $\delta(c, E(c, v), F(c, w))$

## Concrete m1

variables: $a, b, c$	ML_out when $a + b < d$ $c = 0$ then $a := a + 1$ end	ML_in when $c > 0$ then $c := c - 1$ end
invariants: inv1.1: $a \in \mathbb{N}$ inv1.2: $b \in \mathbb{N}$ inv1.3: $c \in \mathbb{N}$ inv1.4: $a + b + c = n$ inv1.5: $a = 0 \vee c = 0$	$H \rightarrow$ $a' = a + 1$ $b' = b$ $c' = c$	$\rightarrow$ $c' = c - 1$ $a' = a$ $b' = b$

effect of  $e$  in the abs. state  $e$   
 effect of  $e$  in con. state  
 $I \dots S$   
 $ML\_out / inv1\_4 / INV$   
 $ML\_in / inv1\_5 / INV$

$d \in \mathbb{N}$  axm0.1  
 $d > 0$  axm0.2  
 $n \in \mathbb{N}$  inv0.1  
 $n \leq d$  inv0.2  
 $a \in \mathbb{N}$   
 $b \in \mathbb{N}$   
 $c \in \mathbb{N}$   
 $a + b + c = n$   
 $a = 0 \vee c = 0$

$d \in \mathbb{N}$  axm0.1  
 $d > 0$  axm0.2  
 $n \in \mathbb{N}$  inv0.1  
 $n \leq d$  inv0.2  
 $a \in \mathbb{N}$   
 $b \in \mathbb{N}$   
 $c \in \mathbb{N}$   
 $a + b + c = n$   
 $a = 0 \vee c = 0$

Q. How many PO/VC rules for model m1?

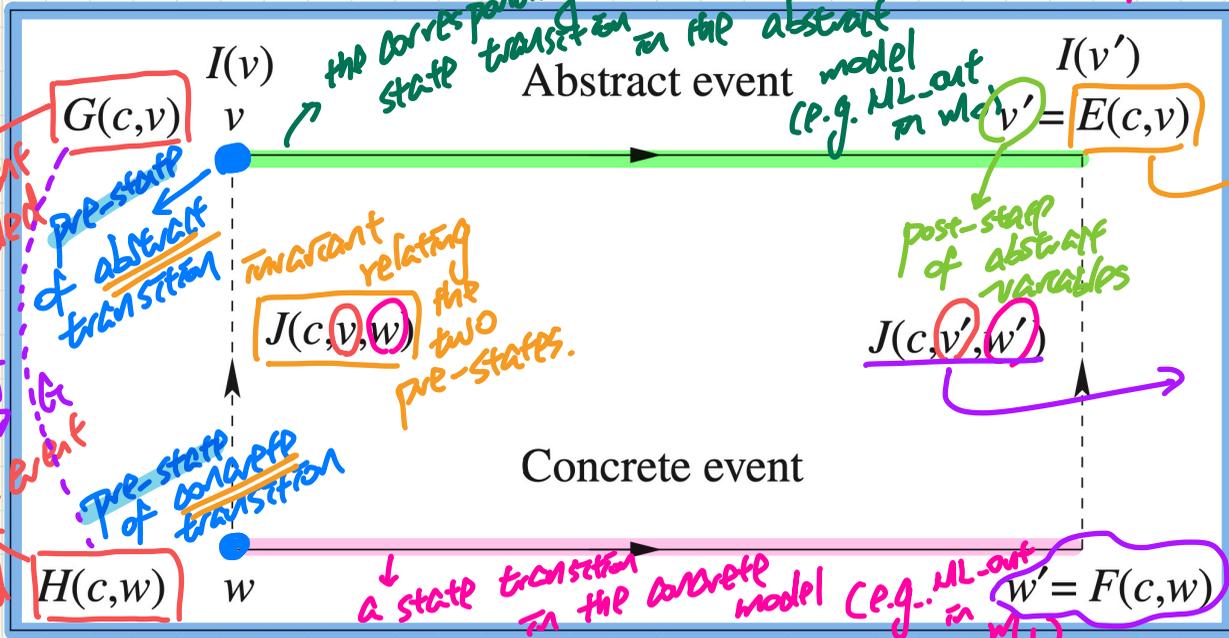
$\vdash$   
 $\vdash$

commuting diagram.

# Visualizing Invariant Preservation in Refinement

Each **concrete state transition** (from  $w$  to  $w'$ ) should be simulated by an **abstract state transition** (from  $v$  to  $v'$ )

$w' = F(c, w)$   
effect of concrete transition  
post-state of concrete variables



time means the obs. event is enabled  
guard strong  
time means the con. event is enabled

the corresponding state transition in the abstract model (e.g. ML-out in  $w'$ )

effect of obs. transition  
same linking inv. holds at the post-state

a state transition in the concrete model (e.g. ML-out in  $w'$ )  
 $w' = F(c, w)$